

Novel Algorithms Based on Legendre Neural Network for Nonlinear Active Noise Control with Nonlinear Secondary Path

Kunal Kumar Das¹ and Jitendriya Kumar Satapathy²

¹ ECE, ITER, SOA University,
² VC, BPUT

Abstract— In this paper, we propose a computationally efficient Legendre Neural Network (LNN) for nonlinear Active Noise Cancellation (NANC). Update algorithms for NANC with linear secondary path (LSP) based on Filtered-x Least Mean Square (FXLMS), Filtered-e Least Mean Square (FELMS) and Recursive Least Square (RLS) are developed. Update algorithm for NANC with nonlinear secondary path (NSP) is also developed which rests upon virtual secondary path concept. Performance of the proposed network and algorithms are validated through extensive computer simulations.

I. INTRODUCTION

Active noise cancellation (ANC) has gained a lot of research interest because of exponential increase of acoustical noise pollution and insufficiency of passive techniques for noise control [1]. ANC uses the superposition principle, where the undesired noise is reduced by adding another noise with the same amplitude but opposite polarity, which is generated by actuators such as loudspeaker [1][2]. The filtered-x LMS algorithm (FXLMS) is the most common algorithm applied in both feedforward and feedback ANC due to its ease in implementation [2]. In the FXLMS algorithm primary path transfer function, $P(z)$, defines the path from the noise source to the cancellation point and $P(n)$ is its impulse response. ANC systems also have secondary path whose transfer function is $S(z)$, which is defined as the path leading from the adaptive filter output to error sensor that measures the residual noise and $S(n)$ is its impulse response. Most available ANC algorithms including FXLMS, require online or offline identification of secondary path. If there is only one reference microphone, one loudspeaker and one error microphone then the situation is termed as a single channel ANC but in case of multichannel ANC more than one reference microphone or loudspeaker or error microphone are present. Several researchers have developed Variations of FXLMS algorithm to improve the canceller performance and robustness

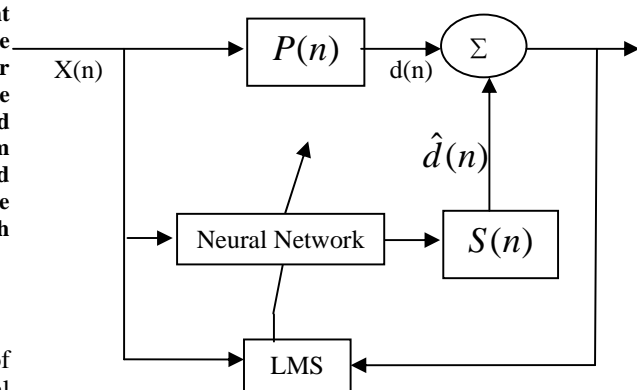


Figure.1 Block diagram of ANC using neural network

In ANC both primary path and secondary path are assumed to be linear but in actual systems primary path and/or secondary path may exhibit nonlinear behavior which results in NANC. Basically algorithms for NANC are classified as NANC with linear secondary path (LSP) and NANC with nonlinear secondary path (NSP). In each of these cases linear adaptive filter performs poorly and in some cases fail to converge. Here we unify the NANC/LSP and NANC/NSP and NANC/LSP is found to be a special case of NANC/NSP.

In [5] Tan and Jiang proposed a NANC using adaptive Volterra filter and developed Volterra FXLMS (VFXLMS) algorithm. In order to reduce the training time of the NN and to improve its convergence rate, an alternate NN structure called functional link NN (FLANN) was proposed by Pao[8]. In [6] Das and Panda employed functional-link artificial neural network (FLANN) to develop filtered-s LMS (FSLMS) algorithm for NANC. In [7] Zhou and DeBrunner generalized the VFXLMS and FSLMS algorithms and extended the algorithms to deal with NANC/NSP. In [3],[4] neural based algorithms are developed. In this paper, we propose a computationally efficient Legendre

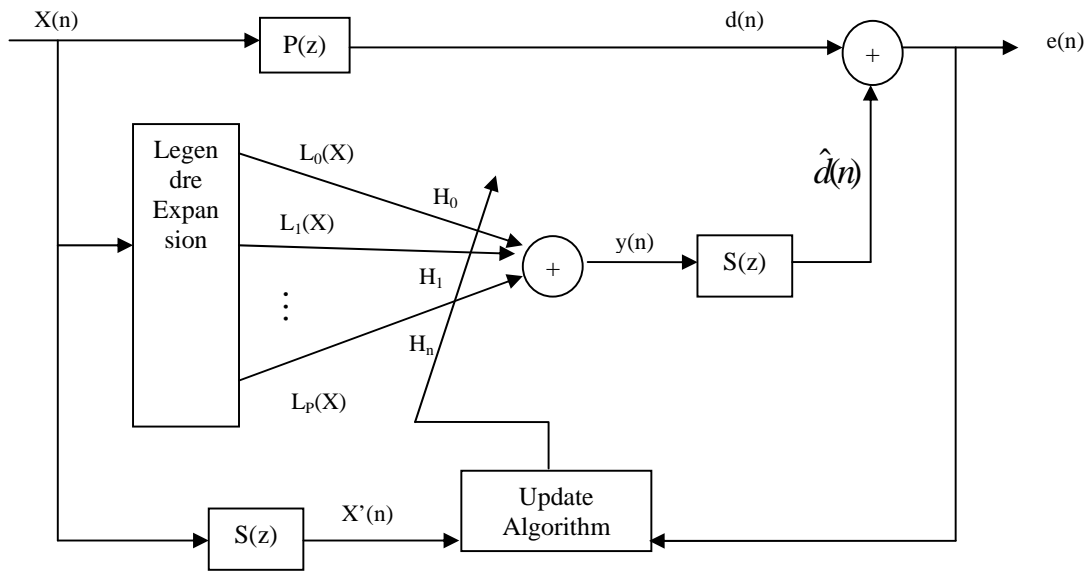


Figure.2 Block diagram of ANC using Legendre neural network

Neural Network (LNN) for nonlinear Active Noise Cancellation(NANC) with nonlinear Secondary path(NSP). Update algorithms based on Filtered-x Least Mean Square (FFLMS), Filtered-e Least Mean Square(FELMS) and Recursive Least Square(RLS) for LNN are first developed. Update algorithm for NANC with NSP is also developed which rests upon virtual secondary path concept.

II. LEGENDER NEURAL NETWORK FOR NANC

The Legendre polynomials are denoted by $L_p(x)$, where P is the order of expansion and $-1 < x < 1$ is the argument of the polynomial. They constitute a set of orthogonal polynomials as solutions to the differential equation

$$\frac{d}{dx} \left[(1-x^2) \frac{dy}{dx} \right] + n(n+1)y = 0.$$

The zero and the first order Legendre polynomials are, respectively, given by $L_0(x) = 1$ and $L_1(x) = x$. The higher order polynomials

$$L_2(x) = \frac{1}{2}(3x^2 - 1)$$

$$L_3(x) = \frac{1}{2}(5x^3 - 3x)$$

$$L_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$$

The recursive formula to generate higher order Legendre polynomials is expressed as

$$L_{p+1}(x) = \frac{1}{(n+1)} [(2n+1)xL_p(x) - nL_{p-1}(x)]$$

Structure of the Legendre NN (LNN) is shown in Fig. 2 where the input vector $X(n) = [x(n) \ x(n-1) \ \dots \ x(n-N+1)]$ is transformed into an output vector $Y(n) = f(X(n))$. The nonlinear function $f(X(n))$ represents a set of the orthogonal basis functions, implemented in the “functional expansion” block. Here the n -dimensional input pattern X is enhanced to an P -dimensional enhanced pattern

$$Y(n) = [L_0(X(n)) \ L_1(X(n)) \ \dots \ L_p(X(n))].$$

Comparing to FLANN [5], in which trigonometric functions are used in the functional expansion, LNN uses Legendre orthogonal functions. The major advantage of LNN over FLANN is that the evaluation of Legendre polynomials involves less computation compared to that of the trigonometric functions. Therefore, LNN offers faster training compared to FLANN. Some of the important properties of Legendre polynomials are that (i) they are orthogonal polynomials, (ii) they arise in numerous problems especially in those involving spheres or spherical coordinates or exhibiting spherical symmetry and (iii) in spherical polar coordinates, the angular dependence is always best handled by spherical harmonics that are defined in terms of Legendre functions. Employing filter bank implementation [5] output of

$$\text{LNN at time } n \text{ is } y(n) = \sum_{i=1}^N y_i(n) = \sum_{i=1}^N L_i(n) H_i(n)$$

Where H_i is the weight vector of i^{th} adaptive filter. Estimated desired signal is obtained by filtering LNN output

by the secondary path $S(z)$. Error at time n is defined

as $e(n) = d(n) - \hat{d}(n)$. Let us define the cost function as $\xi = E[e^2(n)]$. Using the FXLMS algorithm the weight vectors can be updated as $H_i(n+1) = H_i(n) + \mu e(n) X'(n)$ (1)

where $X'(n)$ is the input signal $X(n)$ filtered through the estimated secondary path.

For nonlinear active noise cancellation with nonlinear secondary path (NANC/NSP) defining the virtual secondary path[6]

$$\text{as } \tilde{s}(n) = \begin{bmatrix} \frac{\partial \hat{d}(n)}{\partial u(n)} & \frac{\partial \hat{d}(n)}{\partial u(n-1)} & \frac{\partial \hat{d}(n)}{\partial u(n-2)} & \dots \\ \frac{\partial \hat{d}(n)}{\partial u(n-M)} \end{bmatrix}^T$$

The weight update equation can be written as

$$H_i(n+1) = H_i(n) + \mu e(n) \tilde{X}'(n) \quad (2)$$

where $\tilde{X}'(n)$ is the input signal filtered through virtual secondary path. Saving in computational complexity can be achieved by using the class of filtered error LMS (FELMS) algorithms both for the case of NANC/LSP and NANC/NSP. We define the adjoint secondary path and adjoint virtual secondary path as follows[6]

$$s_-(n) = [s(n, M) \quad s(n, M-1) \quad \dots \quad s(n, 0)]$$

$$s_-(n) = [s(n, M) \quad s(n-1, M-1) \quad \dots \quad s(n-M, 0)]$$

The weight update equation can be written as

$$H_i(n+1) = H_i(n) + \mu e'(n) X(n) \quad (3)$$

Where $e'(n)$ is the error filtered through the adjoint secondary path for NANC /LSP and adjoint virtual secondary path for NANC/NSP. To speed up the convergence RLS algorithm can also be employed. The summary of the LFXRLS algorithm is as follows[1]

$$H_i(n+1) = H_i(n) + \mu e(n) k(n) \quad (4)$$

Where $i = 0, 1, \dots, N+1$

The individual Kalman gain vector

$$k_i(n) = \frac{z_i(n)}{v_i^T(n) z_i(n) + 1}$$

$$z_i(n) = \lambda^{-1} Q_i(n-1) v_i(n)$$

And the inverse of the autocorrelation matrix

$$Q_i(n) = \lambda^{-1} [Q_i(n-1) - k_i(n) z_i(n)]$$

Where $0 \leq \lambda < 1$ is the forgetting factor.

III. SIMULATION RESULTS

Extensive simulation work has been done for various combinations of NANC/NSP and some selected results are presented to validate the proposed algorithm. The performance of the proposed LFXLMS algorithm and LFELMS algorithm for NANC/LSP and NANC/NSP are compared with FLANN based algorithms (FSLMS algorithm).

A. Simulation1

We simulate a NANC/LSP where primary path is modeled as an FIR filter with transfer function

$$P(z) = z^{-5} - 0.3z^{-6} + 0.2z^{-7}$$

The secondary path is a non-minimum-phase FIR filter with transfer function and (perfectly) estimated secondary path transfer function

$$S(z) = z^{-2} + 1.5z^{-3} + z^{-4}$$

The reference noise is the logistic chaotic noise generated

$$\text{by } x(n+1) = \lambda x(n)[1 - x(n)]$$

where $\lambda = 4$ and $x(0) = 0.9$ are used. This noise process is normalized to have unit signal power. MSE plot for proposed LFXLMS, LFELMS and LFXRLS are plotted and compared with the result of FSLMS algorithm. The proposed algorithms results in lower steady state MSE compared to FSLMS algorithm which proves LNN is better than FSLMS algorithm.

B. Simulation2

We simulate NANC/NSP with the primary path defined by the following primary to desired signal relationship

$$\begin{aligned} d(n) = & x(n) + 0.8x(n-1) + 0.3x(n-2) + \\ & 0.4x(n-3) - 0.8x(n)x(n-1) + \\ & 0.9x(n)x(n-2) + 0.7x(n)x(n-3) \end{aligned}$$

NSP has the input to output relationship

$$\begin{aligned} \hat{d}(n) = & y(n) + 0.35y(n-1) + 0.09y(n-2) \\ & - 0.5y(n)y(n-1) + 0.4y(n)y(n-2) \end{aligned}$$

Input is white noise. MSE for the proposed LFXLMS and LFELMS algorithms are obtained using virtual secondary path and compared with FSLMS algorithm. LFXLMS algorithm has faster convergence than FSLMS and LFELMS algorithm

C. Simulation3

In this simulation primary path is defined as

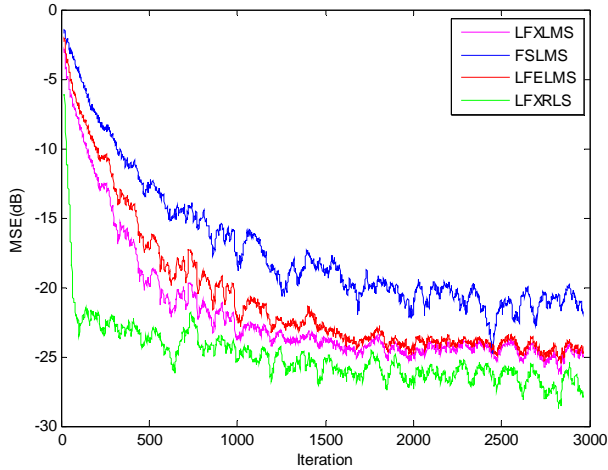


Figure.1 MSE plot for LFXLMS algorithm, FSLMS algorithm, LFELMS algorithm and LFXRLS algorithm

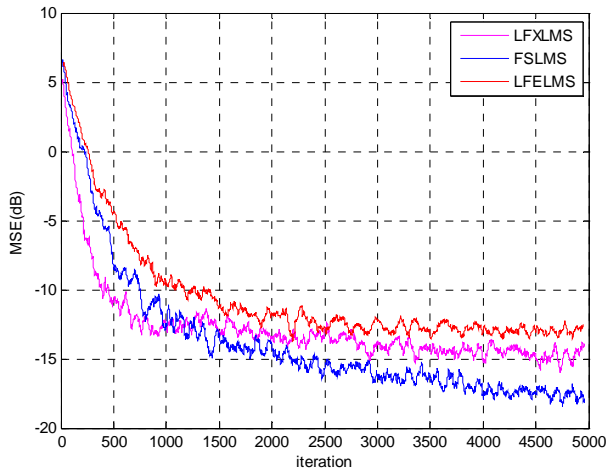


Figure.2 MSE plot for LFXLMS algorithm, FSLMS algorithm and LFELMS algorithm

$$d(n) = x(n - 5) + 0.8x(n - 6) + 0.3x(n - 7) + 0.4x(n - 8) + 0.2x(n - 5)x(n - 6) - 0.3x(n - 5)x(n - 7) + 0.4x(n - 5)x(n - 8)$$

Nonlinear secondary path is same as simulation 3 of [7]. The proposed LFXLMS and LFELMS algorithm show better MSE compared with FSLMS algorithm.

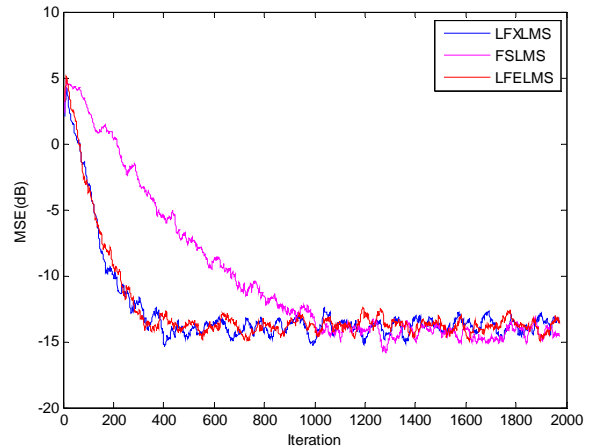


Figure.1 MSE plot for LFXLMS algorithm, FSLMS algorithm and LFELMS algorithm

IV. CONCLUSION

This paper proposes a computationally efficient Legendre Neural Network for Nonlinear active noise cancellation. Update algorithms LFXLMS and LFELMS for the proposed network is derived. RLS algorithm is also employed to get LFXRLS to obtain faster convergence. The extensive simulation demonstrates their better performance compared to FSLMS algorithm.

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